Maximal Covering Problem Model (MCP) for Fire Station Location Determination: A Case Study in Al-Khoms City

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نموذج مشكلة التغطية القصوى (MCP) لتحديد موقع محطة الإطفاء: دراسة حالة في مدينة الخمس

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Abstract
The study addressed the distribution of fire stations in Al-Khoms city, Libya, using a mathematical model for the maximal coverage problem to achieve optimal service coverage with the least number of stations. The model focuses on determining the optimal locations for stations, taking into account the population density and risk levels of the site, and providing a rapid response not exceeding 15 minutes. The study used the simplex method to determine four optimal stations in the areas (Kaam, Al-Khoms power station, Al-Khoms seaport and Ghanima). The results showed improved coverage and reduced 2 stations from the open stations, which are Soq Al-Khames station and Al-Khoms city station. The study recommends establishing a new station in Ghanima, improving roads to support response speed, using GIS systems and updating data continuously. In areas similar to the case study, the model is applicable to achieve the best use of resources.

Keywords: Maximal Covering Problem Model, Fire Station Location, Mathematical Optimization Models, Simplex Method, Response Time.

ملخص
تناولت الدراسة توزيع مراكز الإطفاء في مدينة الخمس بليبيا باستخدام نموذج رياضي لمشكلة التغطية القصوى بهدف تحقيق التغطية المثلى للخدمة بأقل عدد من المحطات. يركز النموذج على تحديد المواقع المثلى للمحطات مع الأخذ في الاعتبار الكثافة السكانية ومستويات المخاطر للموقع وتوفير استجابة سريعة لا تتجاوز 15 دقيقة. استخدمت الدراسة طريقة السمبلكس لتحديد أربع محطات مثالية في المناطق (كعام، محطة كهرباء الخمس، ميناء الخمس البحري وغنيمة). أظهرت النتائج تحسن التغطية وتقليص محطتين من المحطات المفتوحة وهما محطة سوق الخمس ومحطة مدينة الخمس. توصي الدراسة بإنشاء محطة جديدة في غنيمة وتحسين الطرق لدعم سرعة الاستجابة واستخدام أنظمة المعلومات الجغرافية وتحديث البيانات بشكل مستمر. في المناطق المشابهة لدراسة الحالة، يمكن تطبيق النموذج لتحقيق أفضل استخدام للموارد.

الكلمات المفتاحية: نموذج مشكلة التغطية القصوى، موقع محطة الإطفاء، نماذج التحسين الرياضية، طريقة السمبلكس، زمن الاستجابة.

Introduction and Literature Review

The Maximal Coverage Problem represents one of the main challenges in improving the exploitation of limited resources to achieve the maximal possible benefit. In this context, the research aims to present a mathematical model using linear programming to solve this problem by studying the evaluation of the distribution of fire stations in Al-Khoms City/Libya. The model focuses on maximizing the coverage of the demand for the service while taking into account multiple factors, such as population density and the importance of locations. The model was formulated to determine the optimal locations for stations while reducing their number and ensuring a rapid and effective response to emergency cases that does not exceed fifteen minutes, which contributes to improving the management of fire resources and achieving community safety at the lowest cost. Many published papers have addressed the problem of maximal coverage as a solution to many problems, including: (Fang, Kou, Zhou, Zhang, & Yuan, 2023)This paper presents the Maximal Area Coverage (MCR) problem in road networks, which aims to determine the best location for a station so that it can cover the largest number of objects within a specified radius. In this paper, the problem is identified and formulated mathematically, and efficient and accurate solutions are provided using search space reduction techniques as well as proposing approximate solutions that balance efficiency and accuracy. The proposed methods are tested using datasets from real road networks, such as New York and San Francisco, and the proposed solutions have proven their efficiency and effectiveness compared to traditional methods(Imanparast & Kiani, 2021). This paper presents a new heuristic algorithm to solve the problem of maximal location coverage in large continuous spaces. The problem is to locate a given number of facilities to cover as many demand points as possible in a continuous space, which is an NP-Hard problem. A new heuristic algorithm, based on a grid of squares to reduce the search range and identify the best coverage locations, is proposed, and uses a greedy approach to quickly identify optimal circuits. It has been tested on random and real-world data, and has shown accurate results and speed in dealing with large data sets compared to traditional methods(SHAHANDEH & MOSLEHI, 2019).This study addresses the problem of determining the optimal hub coverage problem (Maximum Hub Covering Problem) within dynamic transportation networks that change periodically. The research aims to improve the efficiency of transportation networks by determining the best hub locations to cover the maximum possible flows between the origin and destination points. The model was tested using real data from the CAB dataset, and the results confirmed the efficiency of the solutions in terms of achieving a balance between optimal coverage and costs(Liu, 2022). The paper addresses the topic of improving node coverage in large networks, with a focus on social advertising applications. A genetic algorithm (GA) is used to solve the node coverage problem, where active users are represented as genes, and the goal is to find an optimal set of active users to target with social advertising. A genetic algorithm is designed that involves selection, crossover, and mutation processes to produce better individuals, and the quality of individuals is measured by "fitness" which determines the number of connected neighbors(Reyes-Bedoya, Silva, & Villarceaux, 2013). The research aims to solve two main problems in social networks:

The first is the Maximal Coverage problem: aims to identify the most influential individuals to cover the largest possible number of the target audience.

The second is the Maximal Connected Covering problem: aims to identify the most influential individuals provided that they are connected to ensure strong spread of advertisements. The study found that algorithms that rely on full knowledge of the topology (such as greedy algorithms) perform better compared to partial knowledge, and also in dense graphs, the increased performance of greedy algorithms was evident by 56% to 196% better than random algorithms(Conte, Grossi, & Marino, 2016). The research deals with effective methodologies for covering groups in a large network, where each group is considered a complete branch of the node. Coverage means dividing each node into a few overlapping or non-overlapping groups, and developing algorithms capable of processing large and complex networks while reducing computational complexity using techniques such as (Clique Percolation Method) to understand and analyze advanced real networks. Indeed, algorithms have been developed that have proven their efficiency in covering groups with improved performance compared to traditional methods(Blanquero, Carrizosa, & Boglárka, 2016). The paper deals with a new model of the Maximal Covering Location Problem (MCLP) in networks where the demand is assumed to be continuously distributed over the edges of the network, and this is done by developing an algorithm based on mixed integer nonlinear programming (MINLP) techniques to improve the site selection process. The network is represented as a system containing nodes and edges, where the demand is distributed across the edges(Amiri & Salari, 2019). The study focused on using the Time-Continued Maximum Coverage Problem (TCMCRP) model to improve service coverage within time constraints. This model is an evolution of the Traveling Salesman Problem, where the goal is to achieve maximum customer coverage within a specific time and distance. The study developed a mixed-integer linear programming (MILP) model to solve this problem, in addition to using heuristic algorithms to improve solutions (ILS, TS, VNS). The results showed the effectiveness of the algorithms in reaching high-quality solutions. This model can be applied to emergency healthcare and to determine the locations of field hospitals(Li, Mukhopadhyay, Wu, Zhou, & Du, 2020).The paper discusses a new model called BMCLP (Balanced Maximal Covering Location Problem) which aims to achieve a balance between work locations while maintaining maximum coverage. The proposed model was tested on a bicycle rental system in a large Chinese city, and the results showed significant improvement in distribution and reduction of operational cost. The model can be applied to ambulance, transportation services and logistics services(Idayani, Puspitasari, & Sari, 2021). The research deals with the use of Maximal Covering Problem (MCP) mathematical programming model to determine the optimal location of fire stations in Situbondo area, Indonesia, and the targeted locations are 6 locations, taking into account the time taken to reach and the population density of each area, as well as the weights of the areas and the capacity of the fire station. The study used the Branch and Bound method using the GAMS program to solve the model and reach the optimal solution.

Problem formulation

The Al-Khoms region suffers from challenges in distributing fire stations in a way that achieves optimal coverage of services, especially in the variation of distances between regions. The study aims to develop a mathematical model based on the Maximum Coverage Problem (MCP), so that the coverage of demand for fire services for all regions is maximized with the least possible number of stations. This model is based on formulating an objective function to reduce the number of open stations:

Minimize $\mathbf{Z = \sum_{i=1}^{n} xi}$ Where xi is a binary variable indicating whether the station (i) is open (xi = 1) or closed (xi = 0). This function is restricted by several conditions:

First: Covering all regions within the specified response radius**:** $\sum \mathbf{j} \in \mathbf{Si}\ xj\ \geq \mathbf{1}\ \forall \mathbf{i}$ Where (Si) represents the set of stations that can cover region i.

Second: Response time and distance restrictions: If the distance **dij≤10** km, then station j covers area i

Third: Third: The fire stations located in the Al-Khoms Municipality areas will be evaluated using a model that determines the optimal locations for the stations, taking into account the population density and risk levels for each area, and providing a rapid response not exceeding 15 minutes to ensure coverage of high-priority areas.

The model will be applied to real data from the five areas, with the aim of seeking to provide practical recommendations to improve the efficiency of the fire system and the distribution of its resources.

Case study

This research deals with a case study on the Khoms area, which is located about 135 km east of Tripoli, bordered to the east by the city of Zliten, to the north by the Mediterranean Sea, to the west by the municipality of Qasr Al-Akhyar, to the south by the municipality of Mislata and the municipality of Bani Walid. There are five fire stations in the municipality, distributed as shown in Figure 1. This research aims to address a critical issue of allocating fire stations to achieve optimal coverage of services. The Khoms area is characterized by diverse areas with varying population densities, high-risk industrial sites, and limited firefighting resources. This case study uses real data from the Khoms area, including population statistics, geographical distances, and locations of high-risk areas such as power plants and ports. The aim of the study is to improve service efficiency by redistributing existing stations and adding new stations located in strategic locations. Coverage of all areas with at least one fire station within a radius of 10 km this is on the assumption that the speed of the fire truck is 40 km/hr and the time required to respond is 15 min. Weights will be assigned to areas with high population densities or high-risk levels (such as industrial areas) to give them priority in the improvement process.

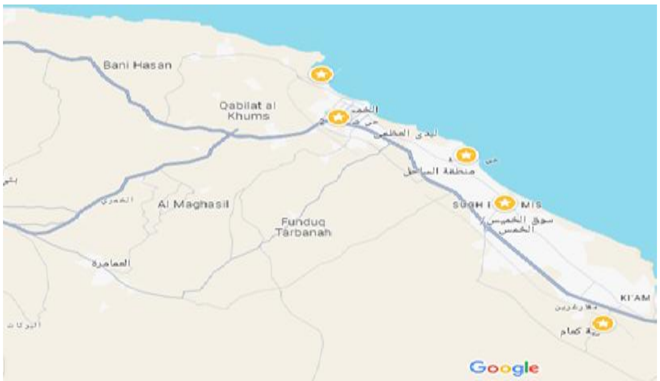

Figure 1. Distribution of fire stations**.**

There are five fire stations distributed across the five regions, except for the Ghanima region, which is located on the western side of the city of Al-Khoms, and for this reason, we will place a virtual fire station there.

The distances between the fire stations targeted by the study are as shown in Table 1.

Table 1 Distribution of distances between the Khoms regions.

		Ghanema X1	Khoms City X2	Soq Al-khames X3	Power station X4	Khoms seaport X5	Kaam X6
X1	Ghanema	0	24	37	33	25	54
X2	Khoms City	24	0	14	12	5	28
X3	Soq Al-khames	37	14	0	8	21	14
X4	Power station	33	12	8	0	13	20
X5	Khoms seaport	25	5	21	13	0	35
X6	Kaam	54	28	14	20	35	0

The distribution of fire stations over the region's geography is as in Figure 2.

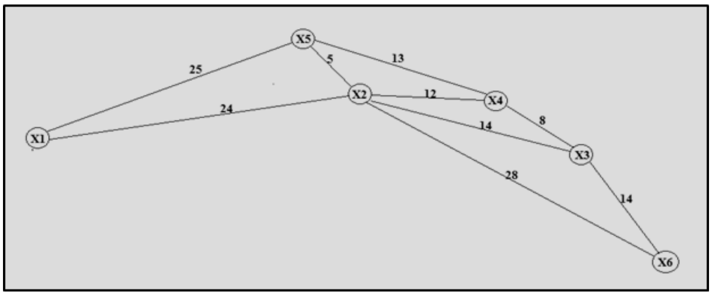

Figure 2 Distribution of fire stations over the geography of the Khoms region.

In this research, the weight is determined by taking into account the population density and industrial areas. The higher the population density, the higher the weight, and the same applies to the risk, as shown in Table 2.

Table 2: Importance (weight) of areas.

Site code	Location Name	Weight (C)	The standard
X1	Ghanema	1	Population
X2	Khoms City	2	Population
X3	Soq Al-khames	2	Population
X4	Power station	3	Dangerous Location
X5	Khoms seaport	3	Dangerous Location
X6	Kaam	1	Population

Min $Z=\sum_{i=1}^{n} ci\,xi \ = c1x1 + c2x2 + c3x3 + c4x4 + c5x5 + c6x6$

Using the simplex method on Excel, we get the results as in Table 3: X1=1, X2=0, X3=0, X4=1, X5=1, X6=1. That is, there will be a station in the Ghanima area, a station in the Khoms seaport, another station at the Khoms power generation station, and finally, a station in the Kaam area**.**

Table 3: Results using the simplex method on Excel.

	X1	X2	X3	X4	X5	X6			
	1	0	0	1	1	1			
Z=	1	2	2	3	3	1	8		
X1	1	0	0	0	0	0	1	<=	1
X2	0	1	0	0	1	0	1	<=	1
X3	0	0	1	1	0	0	1	<=	1
X4	0	0	1	1	0	0	1	<=	1
X5	0	1	0	0	1	0	1	<=	1
X6	0	0	0	0	0	1	1	<=	1

Study results

The mathematical model of the maximum coverage problem was applied to the Al-Khoms area with the aim of redistributing fire stations to provide optimal coverage of services with the least possible number of stations. The results were as follows:

First: Distribution of stations: Determine the locations of the four optimal stations using the simplex method as shown in Figure 3:

- A station in the Kaam area
- A station at the Al-Khoms power plant.
- A station in the Al-Khoms port.
- A station in the Ghanima area.

Second: Improving coverage: Achieving coverage of all areas within a radius of 10 km, taking into account the population density and risk levels in the distribution of weights that reflect the importance of the areas.

Third: Achieving the goal: Reducing the number of open stations and providing a faster response to emergency situations, with a response time not exceeding 15 minutes.

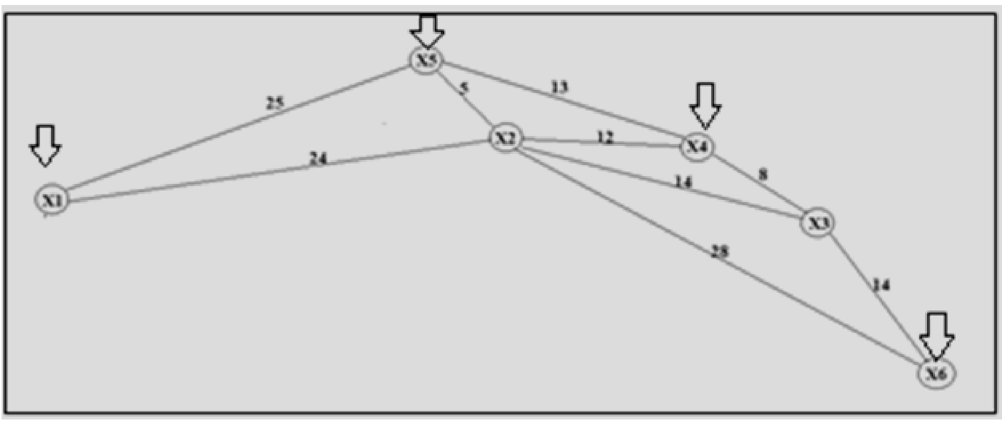

Figure 3 Determining the locations of the four optimal stations.

i. Recommendations

- Establish a fire station in the Ghanima area to cover the geographical gap.

- Use geographic information systems (GIS) to analyze the location and improve the accuracy of the geographical distribution of stations.

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